particulate matter contacts any surface other than the filter, the sample may not be used to determine particulate emissions. In the case of contact with another surface, clean the affected surface before continuing.

- (e) To stabilize PM samples, place them in one or more containers that are open to the PM-stabilization environment, as described in §1065.190. If you expect that a sample medium's (e.g., filter's) total surface concentration of PM will be less than 400 µg, assuming a 38 mm diameter filter stain area, expose the filter to a PM-stabilization environment meeting the specifications of §1065.190 for at least 30 minutes before weighing. If you expect a higher PM concentration or do not know what PM concentration to expect, expose the filter to the stabilization environment for at least 60 minutes before weighing. Note that 400 µg on sample media (e.g., filters) is an approximate net mass of 0.07 g/kW·hr for a hot-start test with compression-ignition engines tested according to 40 CFR part 86, subpart N, or 50 mg/mile for light-duty vehicles tested according to 40 CFR part 86, subpart B.
- (f) Repeat the procedures in §1065.590(f) through (i) to determine post-test mass of the sample media (e.g., filters).
- (g) Subtract each buoyancy-corrected tare mass of the sample medium (e.g., filter) from its respective buoyancy-corrected mass. The result is the net PM mass, $m_{\rm PM}$. Use $m_{\rm PM}$ in emission calculations in § 1065.650.

[73 FR 37323, June 30, 2008]

Subpart G—Calculations and Data Requirements

§ 1065.601 Overview.

- (a) This subpart describes how to—
- (1) Use the signals recorded before, during, and after an emission test to calculate brake-specific emissions of each measured exhaust constituent.

- (2) Perform calculations for calibrations and performance checks.
 - (3) Determine statistical values.
- (b) You may use data from multiple systems to calculate test results for a single emission test, consistent with good engineering judgment. You may also make multiple measurements from a single batch sample, such as multiple weighings of a PM filter or multiple readings from a bag sample. You may not use test results from multiple emission tests to report emissions. We allow weighted means where appropriate. You may discard statistical outliers, but you must report all results.
- (c) You may use any of the following calculations instead of the calculations specified in this subpart G:
- (1) Mass-based emission calculations prescribed by the International Organization for Standardization (ISO), according to ISO 8178, except the following:
- (i) ISO 8178–1 Section 14.4, $NO_{\rm X}$ Correction for Humidity and Temperature. See §1065.670 for approved methods for humidity corrections.
- (ii) ISO 8178-1 Section 15.1, Particulate Correction Factor for Humidity.
- (2) Other calculations that you show are equivalent to within $\pm 0.1\%$ of the brake-specific emission results determined using the calculations specified in this subpart G.

[70 FR 40516, July 13, 2005, as amended at 73 FR 37324, June 30, 2008; 74 FR 56516, Oct. 30, 2009; 75 FR 23044, Apr. 30, 2010]

§ 1065.602 Statistics.

- (a) Overview. This section contains equations and example calculations for statistics that are specified in this part. In this section we use the letter "y" to denote a generic measured quantity, the superscript over-bar "-" to denote an arithmetic mean, and the subscript "ref" to denote the reference quantity being measured.
- (b) Arithmetic mean. Calculate an arithmetic mean, \bar{y} , as follows:

$$\overline{y} = \frac{\sum_{i=1}^{N} y_i}{N}$$
 Eq. 1065.602-1

Example: N = 3

$$y_1 = 10.60$$

 $y_2 = 11.91$
 $y_N = y_3 = 11.09$

$$\overline{y} = \frac{10.60 + 11.91 + 11.09}{3}$$

 $\bar{y} = 11.20$

(c) Standard deviation. Calculate the standard deviation for a non-biased (e.g., N-1) sample, σ , as follows:

N = 3 $y_1 = 10.60$ $y_2 = 11.91$ $y_N = y_3 = 11.09$ $\bar{y} = 11.20$

Example:

$$\sigma_y = \sqrt{\frac{\displaystyle\sum_{i=1}^{N} \left(y_i - \overline{y}\right)^2}{(N-1)}}$$
 Eq. 1065.602-2

$$\sigma_y = \sqrt{\frac{(10.60 - 11.2)^2 + (11.91 - 11.2)^2 + (11.09 - 11.2)^2}{2}}$$

 $\sigma_{y} = 0.6619$

(d) *Root mean square*. Calculate a root mean square, *rms*_y, as follows:

$$rms_y = \sqrt{\frac{1}{N} \sum_{i=1}^{N} {y_i}^2}$$
 Eq. 1065.602-3

Example:

$$N = 3$$

$$y_1 = 10.60$$

$$y_2 = 11.91$$

$$y_{\rm N} = y_3 = 11.09$$

$$rms_y = \sqrt{\frac{10.60 + 11.91^2 + 11.09^2}{3}}$$

 $rms_{v} = 11.21$

(e) Accuracy. Determine accuracy as described in this paragraph (e). Make multiple measurements of a standard quantity to create a set of observed values, y_{i} and compare each observed value to the known value of the standard quantity. The standard quantity may have a single known value, such as a gas standard, or a set of known values of negligible range, such as a known applied pressure produced by a calibration device during repeated applications. The known value of the standard quantity is represented by y_{ref_i} . If you use a standard quantity with a single value, y_{ref_i} would be constant. Calculate an accuracy value as follows:

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$$accuracy = \left| \frac{1}{N} \sum_{i=1}^{N} (y_i - y_{\text{ref}_i}) \right|$$
 Eq. 1065.602-4

$$Example: \\ y_{ref} = 1800.0 \\ N = 3$$

$$y_1 = 1806.4$$

 $y_2 = 1803.1$
 $y_3 = 1798.9$

$$accuracy = \left| \frac{1}{3} ((1806.4 - 1800.0) + (1803.1 - 1800.0) + (1798.9 - 1800.0)) \right|$$

$$accuracy = \left| \frac{1}{3} ((6.4) + (3.1) + (-1.1)) \right|$$

accuracy = 2.8

(f) *t-test*. Determine if your data passes a *t*-test by using the following equations and tables:

(1) For an unpaired t-test, calculate the t statistic and its number of degrees of freedom, v, as follows:

$$t = \frac{\left|\overline{y}_{ref} - \overline{y}\right|}{\sqrt{\frac{\sigma_{ref}^2}{N_{ref}} + \frac{\sigma_y^2}{N}}}$$
Eq. 1065.602-5

$$v = \frac{\left(\frac{\sigma_{ref}^{2}}{N_{ref}} + \frac{\sigma_{y}^{2}}{N}\right)^{2}}{\frac{\left(\sigma_{ref}^{2}/N_{ref}\right)^{2}}{N_{ref} - 1} + \frac{\left(\sigma_{y}^{2}/N\right)^{2}}{N - 1}}$$
Eq. 1065.602-6

Example:

 $\bar{y}_{ref} = 1205.3$ $\bar{y} = 1123.8$ $\sigma_{ref} = 9.399$ $\sigma_{y} = 10.583$ $N_{ref} = 11$

 $N_{\text{ref}} = 11$ N = 7

$$t = \frac{|1205.3 - 1123.8|}{\sqrt{\frac{9.399^2}{11} + \frac{10.583^2}{7}}}$$

v = 11.76

$$t = \frac{|1205.3 - 1123.8|}{\sqrt{\frac{9.399^2}{11} + \frac{10.583^2}{7}}}$$

 $t = 16.63 \\ \sigma_{ref} = 9.399 \\ \sigma_{y} = 10.583 \\ N_{ref} = 11 \\ N = 7$

(2) For a paired t-test, calculate the t statistic and its number of degrees of freedom, v, as follows, noting that the ϵ_i are the errors (e.g., differences) between each pair of y_{refi} and y_i :

$$t = \frac{|\overline{\epsilon}| \cdot \sqrt{N}}{\sigma_{\epsilon}} \qquad \text{Eq. 1065.602-7}$$

Example:

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 $\bar{\epsilon} = -0.12580$ N = 16 $\sigma \epsilon = 0.04837$

$$t = \frac{|-0.12580| \cdot \sqrt{16}}{0.04837}$$

t=10.403v = N - 1Example:

N = 16

v = 16 - 1v = 15

(3) Use Table 1 of this section to compare t to the $t_{\rm crit}$ values tabulated versus the number of degrees of freedom. If t is less than $t_{\rm crit}$, then t passes the t-test. The Microsoft Excel software package contains a TINV function that returns results equivalent to §1065.602 Table 1 and may be used in place of Table 1.

TABLE 1 OF § 1065.602—CRITICAL t VALUES VERSUS NUMBER OF DEGREES OF FREEDOM, v 1

	Confid	lence
v	90%	95%
1	6.314	12.706
2	2.920	4.303
3	2.353	3.182
4	2.132	2.776

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TABLE 1 OF § 1065.602—CRITICAL t VALUES VERSUS NUMBER OF DEGREES OF FREEDOM, ν 1—Continued

	Confid	lence
ν	90%	95%
5	2.015	2.571
6	1.943	2.447
7	1.895	2.365
8	1.860	2.306
9	1.833	2.262
10	1.812	2.228
11	1.796	2.201
12	1.782	2.179
13	1.771	2.160
14	1.761	2.145
15	1.753	2.131
16	1.746	2.120
18	1.734	2.101
20	1.725	2.086
22	1.717	2.074
24	1.711	2.064
26	1.706	2.056
28	1.701	2.048
30	1.697	2.042
35	1.690	2.030
40	1.684	2.021
50	1.676	2.009
70	1.667	1.994
100	1.660	1.984
1000+	1.645	1.960

¹Use linear interpolation to establish values not shown

(g) F-test. Calculate the F statistic as follows:

$$F_y = \frac{\sigma_y^2}{\sigma_{ref}^2}$$
 Eq. 1065.602-8

Example:

$$\sigma_{y} = \sqrt{\frac{\sum_{i=1}^{N} (y_{i} - \overline{y})^{2}}{(N-1)}} = 10.583$$

$$\sigma_{y} = \sqrt{\frac{\sum_{i=1}^{N} (y_{i} - \overline{y})^{2}}{(N-1)}} = 10.583$$

$$\sigma_{ref} = \sqrt{\frac{\sum_{i=1}^{N_{ref}} (y_{refi} - \overline{y}_{ref})^{2}}{(N_{ref} - 1)}} = 9.399$$

$$F = \frac{10.583^2}{9.399^2}$$

F = 1.268

Table 2 of §1065.602–Critical F values, $F_{\rm critoo}$, versus N-1 and $N_{\rm ref}$ -1 at 90 % confidence

(1) For a 90% confidence F-test, use Table 2 of this section to compare F to the $F_{\rm crit90}$ values tabulated versus (N-1) and $(N_{\rm ref}-1)$. If F is less than $F_{\rm crit90}$, then F passes the F-test at 90% confidence.

(2) For a 95% confidence F-test, use Table 3 of this section to compare F to the $F_{\rm crit95}$ values tabulated versus (N-1) and $(N_{\rm ref}-1)$. If F is less than $F_{\rm crit95}$, then F passes the F-test at 95% confidence.

120 1000+	63.06 63.32	9.483 9.491	5.143 5.134	3.775 3.761	3.123 3.105	2.742 2.722	2.493 2.471		2.316 2.293	\dashv	++-		++-+																					
09	62.79 6	9.475 9	5.151 5	3.790	3.140 3	2.762 2	2.514 2	2.339 2	2.208 2		2.107 2	\vdash	+++			 		 	 	 	 	 	 		 							 		
40	62.52	9.466	5.160	3.804	3.157	2.781	2.535	2.361	2.232	2 132	7:13	2.052	2.052	2.052 1.986 1.931	2.132 2.052 1.986 1.931 1.885	2.052 2.052 1.986 1.931 1.885	2.052 2.052 1.986 1.931 1.885 1.845	2.132 2.052 1.986 1.931 1.885 1.845 1.811 1.811	2.052 2.052 1.986 1.931 1.885 1.845 1.811 1.781	2.132 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.781 1.730	2.052 2.052 1.986 1.931 1.885 1.885 1.811 1.781 1.730 1.730	2.052 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.730 1.730	2.052 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.774 1.730 1.708 1.689	2.052 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.730 1.730 1.730 1.689	2.152 2.052 2.052 1.986 1.885 1.885 1.885 1.781 1.781 1.730 1.730 1.730 1.689 1.689 1.671 1.655	2.152 2.052 2.052 1.986 1.885 1.885 1.885 1.881 1.781 1.730 1.730 1.730 1.689 1.689 1.671 1.671 1.655	2.152 2.052 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.730 1.730 1.671 1.671 1.671 1.655	2.152 2.052 1.986 1.931 1.885 1.845 1.811 1.781 1.754 1.730 1.671 1.671 1.671 1.655 1.663	2.11.2 2.05.2 1.986 1.931 1.885 1.845 1.811 1.730 1.730 1.730 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671	2.11.2 2.05.2 2.05.2 2.05.2 1.931 1.885 1.845 1.811 1.734 1.730 1.671 1.	1.152 1.986 1.931 1.885 1.885 1.885 1.784 1.781 1.781 1.781 1.781 1.781 1.671 1.671 1.671 1.673 1.673 1.633	2.11.2 2.11.2 1.986 1.931 1.885 1.885 1.811 1.781 1.781 1.781 1.784 1.671 1.671 1.671 1.671 1.673 1.773 1.774 1.774 1.775	2.11.2 2.11.2 1.986 1.931 1.885 1.845 1.845 1.811 1.781 1.734 1.730 1.671 1.671 1.671 1.671 1.673 1.773 1.774 1.774 1.775	2.11.2 2.11.2 1.986 1.931 1.885 1.845 1.845 1.811 1.781 1.774 1.774 1.774 1.774 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.671 1.673 1.674 1.673 1.674 1.775
30	62.26	9.458	5.168	3.817	3.174	2.800	2.555	2.383	2.255	2.155		2.076	2.076	2.076 2.011 1.958	2.076 2.011 1.958 1.912	2.076 2.011 1.958 1.912 1.873	2.076 2.011 1.958 1.912 1.873 1.839	2.076 2.011 1.958 1.912 1.873 1.839 1.809	2.076 2.011 1.958 1.912 1.873 1.839 1.809 1.783	2.076 2.011 1.958 1.912 1.873 1.839 1.783 1.783	2.076 2.011 1.958 1.912 1.873 1.839 1.783 1.783	2.016 2.011 1.958 1.912 1.839 1.839 1.783 1.783 1.719	2.076 2.011 1.958 1.912 1.839 1.809 1.783 1.783 1.793 1.702	2.076 2.011 1.958 1.912 1.839 1.809 1.759 1.759 1.759 1.769 1.769 1.769 1.769	2.076 2.011 1.958 1.912 1.873 1.839 1.789 1.789 1.79 1.79 1.702 1.702 1.686	2.076 2.011 1.958 1.873 1.839 1.789 1.789 1.779 1.779 1.770 1.686 1.672 1.673	2.076 2.011 1.958 1.912 1.839 1.809 1.703 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713 1.713	2.076 2.011 1.958 1.873 1.839 1.809 1.783 1.739 1.739 1.719 1.702 1.672 1.672 1.673 1.673	2.076 2.011 1.958 1.912 1.873 1.839 1.739 1.739 1.739 1.738 1.739 1.738 1.739 1.740	2.076 2.011 1.958 1.912 1.873 1.839 1.839 1.739 1.759 1.719 1.710 1.686 1.672 1.672 1.673	2.076 2.011 1.958 1.912 1.873 1.839 1.1839 1.783 1.759 1.769 1.702 1.666 1.665 1.665 1.665 1.665	2.076 2.011 1.958 1.912 1.873 1.839 1.783 1.79 1.79 1.702 1.686 1.672 1.659 1.	2.076 2.011 1.958 1.912 1.873 1.839 1.783 1.79 1.702 1.686 1.672 1.686 1.672 1.672 1.636 1.647 1	2.076 2.011 1.958 1.958 1.873 1.839 1.783 1.779
24	62.00	9.450	5.176	3.831	3.191	2.818	2.575	2.404	2.277	2.178		2.100	2.100	2.036	2.036 1.983 1.938	2.100 2.036 1.983 1.938 1.899	2.100 2.036 1.983 1.938 1.899	2.100 2.036 1.983 1.899 1.866 1.836	2.100 2.036 1.938 1.899 1.866 1.836 1.810	2.100 2.036 1.983 1.938 1.899 1.866 1.836 1.810	2.036 2.036 1.938 1.899 1.866 1.836 1.810 1.787	2.100 2.036 1.983 1.899 1.866 1.810 1.810 1.787 1.787	2.100 2.036 1.983 1.989 1.866 1.876 1.787 1.787 1.787	2.100 2.036 1.983 1.899 1.896 1.836 1.836 1.836 1.787 1.787 1.787 1.767	2.100 2.036 1.983 1.899 1.806 1.810 1.810 1.787 1.787 1.787 1.797 1.707	2.100 2.036 1.983 1.938 1.899 1.866 1.810 1.787 1.767 1.748 1.710 1.710	2.100 2.036 1.983 1.938 1.899 1.866 1.810 1.787 1.767 1.748 1.716 1.716 1.702 1.689	2.100 2.036 1.983 1.938 1.899 1.836 1.836 1.836 1.748 1.748 1.748 1.710	2.100 2.036 1.983 1.938 1.836 1.836 1.836 1.787 1.748 1.748 1.716	2.100 2.205 2.036 2.036 1.983 1.899 1.866 1.810 1.787 1.767 1.748 1.716	2.100 2.2036 2.036 1.983 1.983 1.899 1.866 1.810 1.787 1.767 1.716 1.716 1.702 1.666 1.666 1.666	2.100 2.2036 2.036 1.983 1.899 1.806 1.810 1.787 1.767 1.773 1.702 1.669 1.666 1.656 1.656 1.638	2.100 2.036 2.036 1.983 1.899 1.806 1.816 1.787 1.787 1.716 1.702 1.666 1.656 1.656 1.638	2.100 2.036 2.036 1.983 1.899 1.806 1.810 1.787 1.787 1.716 1.716 1.716 1.702 1.689 1.656 1.574 1.647 1.647 1.647 1.647 1.647
07	61.74	9.441	5.184	3.844	3.207	2.836	2.595	2.425	2.298	2.201		2.123	2.123	2.123 2.060 2.007	2.123 2.060 2.007 1.962	2.123 2.060 2.007 1.962 1.924	2.123 2.060 2.007 1.962 1.924 1.891	2.123 2.060 2.007 1.962 1.924 1.891 1.862	2.123 2.060 2.007 1.962 1.924 1.891 1.862 1.862	2.123 2.060 2.007 1.962 1.924 1.891 1.862 1.837	2.020 2.060 2.007 1.962 1.924 1.891 1.862 1.814 1.814	2.123 2.060 2.007 1.962 1.924 1.891 1.862 1.837 1.837 1.814 1.776	2.123 2.000 2.007 1.962 1.891 1.891 1.837 1.837 1.814 1.754 1.756	2.123 2.000 2.007 1.962 1.891 1.891 1.814 1.794 1.776 1.776	2.123 2.060 2.007 1.962 1.891 1.891 1.814 1.76 1.759 1.759 1.739	2.123 2.060 2.007 1.962 1.891 1.891 1.814 1.76 1.75 1.75 1.75 1.74 1.730	2.123 2.060 2.007 1.962 1.924 1.891 1.882 1.837 1.814 1.776 1.776 1.739 1.744 1.730 1.730	2.123 2.060 2.007 1.962 1.924 1.891 1.814 1.776 1.776 1.776 1.739 1.744 1.730 1.744 1.730 1.746 1.730	2.123 2.060 2.060 1.962 1.924 1.891 1.881 1.814 1.794 1.776 1.776 1.739 1.740 1.710	2.123 2.060 2.007 1.962 1.924 1.891 1.883 1.814 1.794 1.776 1.776 1.730 1.730 1.730 1.740 1.740 1.740 1.760	2.123 2.000 2.000 2.007 1.962 1.821 1.831 1.834 1.736 1.736 1.739 1.744 1.744 1.744 1.746 1.769 1.766 1.766 1.766 1.766 1.766 1.766 1.665	2.123 2.000 2.000 2.007 1.962 1.924 1.821 1.837 1.837 1.814 1.736	2.123 2.000 2.000 2.000 1.924 1.891 1.891 1.882 1.893 1.706 1.704 1.704 1.706 1.706 1.685 1.685 1.685 1.685 1.685	2.123 2.000 2.000 2.000 1.9024 1.891 1.8921 1.8837 1.814 1.776 1.7
15	61.22	9.425	5.200	3.870	3.238	2.871	2.632	2.464	2.340	2.244		2.167	2.167	2.167	2.167 2.105 2.053 2.010	2.167 2.053 2.010 2.010 1.972	2.167 2.053 2.010 1.972 1.940	2.167 2.053 2.010 2.010 1.972 1.940	2.167 2.105 2.053 2.010 1.972 1.940 1.912	2.167 2.105 2.053 2.010 1.972 1.940 1.912 1.887	2.167 2.105 2.003 2.010 1.972 1.940 1.912 1.887 1.865	2.167 2.105 2.010 2.010 1.972 1.940 1.912 1.887 1.865 1.845	2.105 2.105 2.010 1.972 1.940 1.912 1.887 1.885 1.845 1.845 1.845	2.167 2.105 2.010 2.010 1.940 1.912 1.887 1.865 1.845 1.845 1.827 1.811	2.167 2.105 2.053 2.010 1.972 1.940 1.887 1.887 1.845	2.167 2.105 2.053 2.010 1.972 1.940 1.887 1.887 1.845	2.167 2.105 2.003 2.000 1.972 1.940 1.912 1.887 1.865 1.845	2.167 2.105 2.003 2.001 1.972 1.940 1.912 1.887 1.885 1.885 1.887 1.877 1.811 1.796 1.796 1.796 1.796 1.796	2.167 2.105 2.053 2.010 1.972 1.1940 1.1940 1.887 1.887 1.887 1.887 1.887 1.887 1.887 1.817 1.817 1.716 1.771 1.771 1.771 1.771 1.771 1.773	2.167 2.105 2.053 2.010 1.972 1.940 1.1940 1.887 1.887 1.887 1.887 1.887 1.887 1.817 1.716 1.771 1.771 1.771 1.773	2.167 2.105 2.003 2.003 2.010 1.940 1.912 1.887 1.885 1.845 1.845 1.845 1.845 1.845 1.740 1.740 1.740 1.740	2.167 2.105 2.105 2.203 2.2010 1.972 1.940 1.887 1.887 1.885 1.885 1.885 1.885 1.885 1.885 1.885 1.796 1.796 1.796 1.791 1.711	2.107 2.105 2.105 2.083 2.083 2.083 2.091 1.970 1.912 1.845 1.845 1.845 1.845 1.771 1.771 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770	2.107 2.105 2.105 2.105 2.083 2.083 2.010 2.010 1.972 1.887 1.887 1.884 1.887 1.887 1.788 1.789 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770 1.770
12	60.70	9.408	5.216	3.896	3.268	2.905	2.668	2.502	2.379	2.284	0000	2.202	2.147	2.147	2.147 2.097 2.054	2.209 2.097 2.097 2.054 2.017	2.147 2.097 2.054 2.017 1.985	2.147 2.097 2.054 2.017 1.985 1.958	2.2097 2.097 2.097 2.017 1.985 1.958	2.147 2.097 2.097 2.054 2.017 1.985 1.933 1.933	2.147 2.097 2.097 2.017 1.985 1.985 1.933 1.912 1.912	2.147 2.097 2.0054 2.017 1.985 1.938 1.912 1.912 1.892	2.147 2.097 2.017 2.017 1.985 1.933 1.912 1.892 1.875	2.147 2.097 2.097 2.017 1.985 1.985 1.933 1.892 1.875 1.875 1.875	2.147 2.097 2.097 2.017 1.985 1.912 1.912 1.815 1.845 1.835	2.147 2.097 2.097 2.017 1.985 1.912 1.912 1.875 1.859 1.835 1.835	2.147 2.097 2.097 2.017 1.985 1.933 1.912 1.892 1.875 1.875 1.839 1.830 1.830	2.147 2.097 2.007 2.017 1.985 1.985 1.912 1.815 1.826 1.836	2.147 2.034 2.017 2.017 1.988 1.913 1.912 1.892 1.875	2.147 2.054 2.017 2.017 1.988 1.913 1.912 1.892 1.875	2.207 2.017 2.017 2.017 1.985 1.912 1.912 1.875	2.207 2.097 2.057 2.017 1.985 1.933 1.933 1.933 1.875 1.875 1.820 1.820 1.820 1.731 1.731	2.074 2.074 2.077 1.985 1.938 1.938 1.938 1.875	2.074 2.074 2.077 1.985 1.938 1.938 1.938 1.875
10	60.19	9.392	5.230	3.920	3.297	2.937	2.703	2.538	2.416	2.323	3 2 48	2.740	2.188	2.188	2.188	2.138 2.095 2.059	2.188 2.138 2.095 2.059 2.028	2.188 2.138 2.095 2.059 2.028	2.138 2.095 2.095 2.059 2.028 2.001 1.977	2.188 2.138 2.095 2.059 2.028 2.001 1.977	2.188 2.138 2.095 2.095 2.028 2.001 1.977 1.956	2.188 2.138 2.095 2.059 2.028 2.001 1.977 1.956 1.956	2.188 2.188 2.095 2.005 2.001 2.001 1.977 1.956 1.937	2.188 2.138 2.095 2.005 2.001 1.977 1.976 1.920 1.904	2.188 2.138 2.005 2.005 2.002 2.001 1.977 1.956 1.956 1.950 1.950 1.950	2.188 2.138 2.005 2.005 2.001 1.977 1.956 1.937 1.920 1.930 1.890 1.890	2.138 2.138 2.005 2.005 2.002 2.001 1.977 1.926 1.937 1.920 1.930 1.866 1.885	2.138 2.039 2.005 2.005 2.002 2.001 1.977 1.956 1.937 1.920 1.800 1.804 1.805 1.845	2.188 2.188 2.013 2.005 2.005 2.001 1.977 1.956 1.930 1.800 1.845 1.845 1.836	2.188 2.188 2.038 2.009 2.009 2.001 1.977 1.956 1.930 1.800 1.845 1.835 1.835 1.835 1.835	2.138 2.095 2.095 2.009 2.001 1.977 1.936 1.930 1.890 1.845 1.845 1.846 1.836 1.836 1.836	2.188 2.005 2.005 2.005 2.001 1.977 1.920 1.920 1.920 1.845 1.845 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836 1.836	2.138 2.138 2.059 2.028 2.020 1.956 1.956 1.950	2.138 2.138 2.059 2.028 2.028 2.001 1.956 1.956 1.950 1.950 1.950 1.870 1.870 1.871
6	59.85	9.381	5.240	3.936	3.316	2.958	2.725	2.561	2.440	2.347	2774	417.7	2.214	2.214	2.214	2.214 2.164 2.122 2.086	2.214 2.164 2.122 2.086 2.085	2.214 2.164 2.122 2.086 2.085 2.028	2.214 2.164 2.162 2.086 2.055 2.028 2.028	2.214 2.164 2.164 2.122 2.086 2.055 2.005 2.005	2.214 2.214 2.164 2.122 2.086 2.055 2.028 2.005 1.984 1.984	2.214 2.164 2.164 2.086 2.085 2.028 2.005 1.984 1.984	2.214 2.164 2.164 2.086 2.085 2.028 2.005 2.005 1.984 1.965 1.965	2.214 2.214 2.164 2.122 2.086 2.055 2.028 2.005 1.984 1.984 1.965 1.948	2.214 2.214 2.164 2.122 2.086 2.055 2.028 2.005 1.984 1.984 1.948 1.933 1.919	2.214 2.2164 2.2162 2.086 2.085 2.028 2.005 1.984 1.984 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948	2.214 2.214 2.2164 2.162 2.086 2.085 2.028 2.005 1.984 1.965 1.948 1.919 1.919 1.906 1.884	2.214 2.122 2.122 2.086 2.055 2.005 2.005 1.984 1.919 1.919 1.906 1.895 1.884 1.874	2.214 2.214 2.122 2.086 2.085 2.005 2.005 1.984 1.948 1.919 1.919 1.919 1.919 1.919 1.919 1.919 1.919 1.919 1.919 1.919 1.919	2.114 2.124 2.124 2.125 2.085 2.028 2.005 1.984 1.984 1.919 1.906 1.895 1.895 1.865 1.865	2.214 2.124 2.125 2.086 2.035 2.038 2.008 1.984 1.995 1.906 1.919 1.919 1.919 1.919 1.814 1.814 1.865	2.114 2.164 2.125 2.086 2.005 2.005 2.005 1.984 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.948 1.849	2.124 2.124 2.125 2.038 2.035 2.038 2.038 1.984 1.984 1.996 1.996 1.996 1.998	2.124 2.124 2.125 2.036 2.035 2.035 2.038 1.984 1.984 1.995 1.996
∞	59.43	9.367	5.252	3.955	3.339	2.983	2.752	2.589	2.469	2.377	2304	4.304	2.245	2.245	2.245	2.245 2.195 2.195 2.154 2.119	2.245 2.195 2.195 2.154 2.119 2.088	2.245 2.245 2.195 2.119 2.119 2.088	2.245 2.195 2.195 2.119 2.088 2.061 2.038	2.245 2.195 2.195 2.195 2.119 2.088 2.061 2.038	2.245 2.245 2.195 2.119 2.088 2.081 2.001 2.017	2.245 2.245 2.195 2.119 2.081 2.061 2.017 2.017 1.999	2.245 2.245 2.195 2.119 2.088 2.081 2.081 2.017 1.999 1.999	2.245 2.245 2.195 2.119 2.019 2.001 2.017 1.999 1.982 1.982 1.983	2.2245 2.2245 2.195 2.119 2.011 2.038 2.017 1.999 1.999 1.982 1.967 1.967	2.304 2.2245 2.195 2.119 2.081 2.061 2.038 2.017 1.999 1.999 1.982 1.967 1.967 1.967	2.245 2.245 2.2154 2.119 2.218 2.001 2.001 2.003 2.017 1.999 1.982 1.967 1.967 1.967 1.967 1.967 1.967	2.245 2.245 2.2154 2.119 2.218 2.001 2.003 2.001 1.999 1.999 1.990 1.967 1.967 1.967 1.967 1.967 1.967 1.967	2.245 2.245 2.195 2.119 2.018 2.061 2.038 2.017 1.999 1.982 1.982 1.967 1.973	2.245 2.245 2.195 2.119 2.018 2.061 2.038 2.017 1.999 1.982 1.941 1.929 1.919 1.919 1.919	2.245 2.245 2.195 2.195 2.088 2.061 2.038 2.017 1.999 1.982 1.953 1.919 1.919 1.900 1.900	2.195 2.195 2.119 2.119 2.088 2.088 2.088 2.088 2.081 2.081 1.999	2.234 2.195 2.119 2.119 2.017 2.017 2.017 1.999	2.245 2.195 2.119 2.119 2.017 2.017 2.017 1.999
7	58.90	9.349	5.266	3.979	3.368	3.014	2.785	2.624	2.505	2.414	2.342	-	2.283	2.283	2.283	2.283 2.234 2.193 2.158	2.283 2.234 2.193 2.158 2.128	2.283 2.234 2.193 2.158 2.128 2.102	2.283 2.234 2.193 2.158 2.128 2.102 2.002	2.283 2.234 2.193 2.158 2.128 2.102 2.079 2.079	2.283 2.234 2.193 2.158 2.128 2.102 2.079 2.079 2.058	2.283 2.234 2.193 2.158 2.128 2.102 2.079 2.079 2.040	2.283 2.234 2.193 2.158 2.102 2.009 2.040 2.040 2.040 2.023	2.283 2.234 2.193 2.158 2.102 2.079 2.058 2.040 2.023 2.023 2.023	2.283 2.234 2.193 2.158 2.102 2.079 2.040 2.040 2.040 2.008 1.995 1.983	2.283 2.234 2.193 2.158 2.102 2.079 2.040 2.040 2.040 2.040 2.008 1.995 1.995	2.283 2.234 2.193 2.158 2.102 2.079 2.079 2.079 2.040 2.040 2.023 2.040 1.995 1.995 1.983	2.283 2.234 2.193 2.158 2.102 2.009 2.008 2.008 2.008 1.995 1.995 1.991 1.961	2.283 2.234 2.193 2.102 2.079 2.040 2.023 2.008 2.008 1.995 1.995 1.995 1.995 1.995	2.283 2.234 2.193 2.102 2.079 2.040 2.023 2.023 2.008 1.995 1.995 1.995 1.995 1.995	2.283 2.234 2.193 2.102 2.102 2.079 2.040 2.023 2.040 2.023 2.040 1.995 1.995 1.991 1.991 1.991 1.991 1.991 1.991 1.993 1.993	2.283 2.234 2.193 2.102 2.102 2.003 2.003 2.004 2.003 2.003 2.003 2.003 2.003 1.995 1.995 1.995 1.995 1.995 1.993 1.993 1.993	2.283 2.293 2.193 2.118 2.102 2.079 2.079 2.079 2.083 2.083 2.083 2.093	2.283 2.213 2.193 2.118 2.102 2.079 2.079 2.040 2.040 1.943 1.943 1.941
•	58.20	9.326	5.285	4.010	3.405	3.055	2.827	2.668	2.551	2.461	2.389	-	2.331	2.331	2.331	2.283 2.243 2.208	2.283 2.283 2.243 2.208 2.178	2.331 2.283 2.243 2.208 2.178 2.178	2.331 2.283 2.243 2.208 2.178 2.152 2.152	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.130	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.109	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.109 2.091	2.331 2.283 2.243 2.208 2.178 2.152 2.152 2.190 2.091 2.091	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.109 2.001 2.075 2.047	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.109 2.091 2.091 2.075 2.075 2.047	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.109 2.091 2.075 2.075 2.075 2.077 2.075 2.077 2.077	2.331 2.283 2.243 2.248 2.178 2.152 2.130 2.091 2.091 2.097 2.097 2.093 2.024 2.035	2.331 2.283 2.243 2.208 2.178 2.152 2.109 2.091 2.075 2.047 2.047 2.035 2.024 2.036	2.331 2.283 2.243 2.208 2.178 2.152 2.130 2.091 2.075 2.047 2.047 2.047 2.035 2.034 2.034 2.035 2.034 2.037	2.331 2.283 2.243 2.208 2.178 2.152 2.190 2.091 2.001 2.004 2.005 2.004 2.005 2.004 2.005 2.004 2.005 2.005 2.005 2.001	2.331 2.283 2.243 2.208 2.178 2.152 2.109 2.091 2.091 2.097 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007 2.007	2.331 2.243 2.243 2.243 2.178 2.178 2.130 2.091 2.075 2.075 2.047 2.047 2.047 2.047 2.047 2.047 2.047 2.047 2.047 2.047 2.047 2.088 2.014	2.331 2.243 2.243 2.208 2.178 2.190 2.005 2.007	2.331 2.243 2.208 2.178 2.190 2.190 2.005 2.005 2.007
2	57.24	9.293	5.309	4.051	3.453	3.108	2.883	2.726	2.611	2.522	2.451	-	2.394	2.394	2.394	2.394 2.347 2.307 2.273	2.394 2.347 2.307 2.273 2.244	2.394 2.347 2.307 2.273 2.244 2.218	2.394 2.347 2.273 2.274 2.218 2.196	2.394 2.347 2.307 2.273 2.244 2.218 2.196 2.176	2.394 2.347 2.273 2.274 2.218 2.196 2.176 2.176	2.394 2.347 2.273 2.273 2.244 2.218 2.196 2.176 2.176 2.176	2.394 2.347 2.273 2.274 2.218 2.196 2.158 2.142 2.142	2.394 2.347 2.273 2.244 2.218 2.196 2.176 2.176 2.178 2.142 2.128 2.128	2.394 2.347 2.273 2.244 2.218 2.196 2.176 2.178 2.158 2.142 2.128 2.113 2.113	2.394 2.347 2.273 2.273 2.274 2.218 2.196 2.176 2.176 2.178 2.128 2.128 2.115 2.103 2.103	2.394 2.347 2.273 2.274 2.218 2.196 2.106 2.116 2.118 2.103 2.103 2.092 2.082	2.394 2.347 2.273 2.274 2.218 2.218 2.176 2.176 2.178 2.178 2.118 2.115 2.103 2.092 2.082 2.083	2.394 2.307 2.273 2.244 2.218 2.196 2.168 2.168 2.142 2.142 2.118 2.103 2.092 2.092 2.003	2.394 2.347 2.273 2.273 2.244 2.218 2.196 2.176 2.178 2.128 2.103 2.092 2.092 2.093 2.064	2.394 2.347 2.273 2.273 2.244 2.218 2.196 2.176 2.178 2.118 2.103 2.092 2.092 2.082 2.082 2.064 2.049	2.394 2.347 2.273 2.273 2.244 2.218 2.196 2.196 2.198 2.198 2.198 2.108 2.103 2.092 2.003 2.004 2.004 2.004 2.004 2.004 2.007	2.394 2.347 2.244 2.218 2.218 2.196 2.118 2.118 2.118 2.118 2.103	2.394 2.347 2.2307 2.218 2.218 2.2196 2.116 2.118 2.116 2.118 2.116 2.118 2.103 2.103 2.103 2.103 2.103 2.103 2.103 2.203 2.004 2.004 2.007 2.004 2.007 2.00
4	55.83	9.243	5.343	4.107	3.520	3.181	2.961	2.806	2.693	2.605	2.536		2.480	2.434	2.434 2.395	2.434 2.395 2.361	2.480 2.434 2.395 2.361 2.333	2.480 2.434 2.395 2.361 2.333 2.308	2.480 2.434 2.395 2.361 2.333 2.308 2.286	2.480 2.434 2.395 2.361 2.333 2.286 2.286	2.480 2.434 2.395 2.361 2.361 2.308 2.286 2.266	2.480 2.434 2.395 2.361 2.308 2.308 2.266 2.266 2.249	2.480 2.434 2.395 2.361 2.308 2.286 2.266 2.266 2.249 2.233 2.233	2.480 2.434 2.395 2.361 2.308 2.286 2.266 2.266 2.249 2.233 2.219 2.207	2.480 2.434 2.395 2.361 2.308 2.286 2.266 2.249 2.249 2.219 2.219 2.219	2.480 2.434 2.395 2.361 2.333 2.238 2.266 2.266 2.249 2.219 2.207 2.195 2.195	2.480 2.395 2.361 2.333 2.286 2.286 2.286 2.266 2.249 2.219 2.207 2.195 2.195 2.194 2.174	2.480 2.395 2.361 2.308 2.208 2.249 2.207 2.207 2.207 2.219 2.207 2.219 2.219 2.219 2.219 2.219 2.219 2.219 2.219 2.219	2.480 2.395 2.361 2.308 2.308 2.286 2.266 2.249 2.219 2.207 2.219	2.480 2.434 2.395 2.361 2.308 2.206 2.249 2.219 2.207 2.219	2.480 2.434 2.305 2.305 2.308 2.206 2.206 2.207 2.207 2.105	2.480 2.434 2.395 2.361 2.386 2.286 2.286 2.249 2.233 2.219 2.207 2.195 2.194 2.165	2.480 2.434 2.395 2.308 2.208 2.207 2.219 2.207 2.195 2.195 2.165	2.480 2.434 2.395 2.308 2.286 2.286 2.249 2.219 2.207 2.195 2.165
3	53.59	9.162	5.391	4.191	3.619	3.289	3.074	2.924	2.813	2.728	2.660		2.606	2.606	2.606 2.560 2.522				 	 	 	 	 	 	 		 		┞╶┞┈┞┈┞┈┞┈┞┈┞┈┞┈┞┈┞┈	┞╶╏┈╏┈╏┈╏┈╏┈╏┈╏┈╏┈╏ ┈╏	┞┞╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸	╒╏┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋	┞┞╏╸╏╸╏╸╏╸╏╸╏╸╏╸╏╸	┞╴╏╶╏╶╏╶╏╶╏╶╏╶╏╶╏
2	49.50	9.000	5.462	4.325	3.780	3.463	3.257	3.113	3.006	2.924	2.860		2.807	2.763	2.807 2.763 2.726	 			 	 	 - - - - - - - - - - - - - - - - - - -	 	 	┞╶┞┈┞┈┞┈┞┈┞┈┞┈┞┈┞┈			┞╒┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋	┞╃╄╃╇╇╇╇	┞╃┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋	┞┞┞╒┋┋┋┋┋	┞┞┞┞┞╏╏╏╏╏	┞┞┞╒┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋┋	┞┦╃┡╃╃╃╃╃╃╃╃╃╃╃╃╇	┞┞╒┡╒┋╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒╒
-	39.86	8.526	5.538	4.545	4.060	3.776	3.589	3.458	3.360	3.285	3.225		3.177	_	-	- - - 					- - - - - - - - - - 	├─┼─┼─┼─┼─┼			╒┩┩┩ ═ ╏╸┢╶╂╶┩╶┩╶┦╶┦╶┦	╒╃╃╃╃	╒╃╒╃╒╏╒╏╸╏╸╏╸╏╶╏╶╏╶╏╶╏	╒╃╃┩┩┍┡╫┼╃╃╃┩	╒┪┩┩╒╏┩╏╏┩╏┩╏ ╇	╒╃╃╃╃╘ ┼┼┼┼┼┼┼┼┼┼┼┼┼┼	╒╃╃┩┩╒╏╒╏┍ ╫┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼	╒╃╃┩ ╀╃╃╃╃╃╃╃╃╃╃╃	╶┩ ┩ ┩╒┩┩	╒┪┩┩┩ ┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼┼
N-1 Nref-1	-	2	3	4	5	9	4	8	6	10	11		12	12	13	12 13 14 15	12 13 14 15 16	12 13 14 15 16	12 13 14 15 16 17	12 13 14 15 16 17 18	12 13 14 15 16 17 17 18	12 13 14 15 16 17 17 18 19 20 20	112 113 114 116 117 118 119 20 20 20	112 113 114 115 116 117 118 119 20 20 20 20 20 23	112 113 114 115 116 117 119 20 20 20 20 23	12 13 14 16 16 17 17 18 19 20 20 21 21 23 23								

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Table 3 of §1065.602–Critical F values, $F_{\rm cri055}$, versus N-1 and $N_{\rm re}\Gamma 1$ at 95 % confidence

N-1	-	2	3	4	5	9	7	8	6	10	12	15	20	24	30	40	99	120	1000+
N _{ref} 1																			
	161.4	5'661	215.7	224.5	230.1	233.9	236.7	238.8	240.5	241.8	243.9	245.9	248.0	249.0	250.1	251.1	252.2	253.2	254.3
2	18.51	19.00	19.16	19.24	19.29	19.33	19.35	19.37	19.38	19.39	19.41	19.42	19.44	19.45	19.46	19.47	19.47	19.48	19.49
3	10.12	9.552	9.277	9.117	9.014	8.941	8.887	8.845	8.812	8.786	8.745	8.703	8.660	8.639	8.617	8.594	8.572	8.549	8.526
4	7.709	6.944	165.9	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.858	5.803	5.774	5.746	5.717	5.688	5.658	5.628
2	809.9	5.786	5.410	5.192	5.050	4.950	4.876	4.818	4.773	4.735	4.678	4.619	4.558	4.527	4.496	4.464	4.431	4.399	4.365
9	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.938	3.874	3.842	3.808	3.774	3.740	3.705	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.511	3.445	3.411	3.376	3.340	3.304	3.267	3.230
∞	5.318	4.459	4.066	3.838	3.688	3.581	3.501	3.438	3.388	3.347	3.284	3.218	3.150	3.115	3.079	3.043	3.005	2.967	2.928
6	5.117	4.257	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	3.006	2.937	2.901	2.864	2.826	2.787	2.748	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.136	3.072	3.020	2.978	2.913	2.845	2.774	2.737	2.700	2.661	2.621	2.580	2.538
=	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.719	2.646	2.609	2.571	2.531	2.490	2.448	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.617	2.544	2.506	2.466	2.426	2.384	2.341	2.296
53	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.533	2.459	2.420	2.380	2.339	2.297	2.252	2.206
4	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.463	2.388	2.349	2.308	2.266	2.223	2.178	2.131
15	4.543	3.682	3.287	3.056	2.901	2.791	2.707	2.641	2.588	2.544	2.475	2.403	2.328	2.288	2.247	2.204	2.160	2.114	2.066
16	4.494	3.634	3.239	3.007	2:852	2.741	2.657	2.591	2.538	2.494	2.425	2.352	2.276	2.235	2.194	2.151	2.106	2.059	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.308	2.230	2.190	2.148	2.104	2.058	2.011	1.960
81	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.269	2.191	2.150	2.107	2.063	2.017	1.968	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.234	2.156	2.114	2.071	2.026	1.980	1.930	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.203	2.124	2.083	2.039	1.994	1.946	1.896	1.843
21	4.325	3.467	3.073	2.840	2.685	2.573	2.488	2.421	2.366	2.321	2.250	2.176	2.096	2.054	2.010	1.965	1.917	1.866	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.151	2.071	2.028	1.984	1.938	1.889	1.838	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.128	2.048	2.005	1.961	1.914	1.865	1.813	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	2.108	2.027	1.984	1.939	1.892	1.842	1.790	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.237	2.165	2.089	2.008	1.964	1.919	1.872	1.822	1.768	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.266	2.220	2.148	2.072	1.990	1.946	1.901	1.853	1.803	1.749	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	2.056	1.974	1.930	1.884	1.836	1.785	1.731	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	2.041	1.959	1.915	1.869	1.820	1.769	1.714	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.105	2.028	1.945	1.901	1.854	1.806	1.754	1.698	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.092	2.015	1.932	1.887	1.841	1.792	1.740	1.684	1.622
40	4.085	3.232	2.839	2.606	2.450	2.336	2.249	2.180	2.124	2.077	2.004	1.925	1.839	1.793	1.744	1.693	1.637	1.577	1.509
09	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993	1.917	1.836	1.748	1.700	1.649	1.594	1.534	1.467	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.959	1.911	1.834	1.751	1.659	1.608	1.554	1.495	1.429	1.352	1.254
1000+	3.842	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.752	1.666	1.571	1.517	1.459	1.394	1.318	1.221	1.000

(h) Slope. Calculate a least-squares regression slope, $a_{\rm ly}$, as follows:

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$$a_{1y} = \frac{\sum_{i=1}^{N} (y_i - \overline{y}) \cdot (y_{refi} - \overline{y}_{ref})}{\sum_{i=1}^{N} (y_{refi} - \overline{y}_{ref})^2}$$
 Eq. 1065.602-9

Example: N = 6000 $y_1 = 2045.8$

 $\bar{y} = 1051.1$ $y_{\text{ref 1}} = 2045.0$ $\bar{y}_{\text{ref}} = 1055.3$

$$a_{iy} = \frac{\left(2045.8 - 1050.1\right) \cdot \left(2045.0 - 1055.3\right) + \dots + \left(y_{6000} - 1050.1\right) \cdot \left(y_{ref6000} - 1055.3\right)}{\left(2045.0 - 1055.3\right)^2 + \dots + \left(y_{ref6000} - 1055.3\right)^2}$$

 $a_{1y} = 1.0110$

(i) *Intercept*. Calculate a least-squares regression intercept, a_{0y} , as follows:

$$a_{0y} = \overline{y} - (a_{1y} \cdot \overline{y}_{ref})$$
 Eq. 1065.602-10

Example:

 $\bar{y} = 1050.1$ $\alpha_{1y} = 1.0110$

 $\bar{y}_{\rm ref} = 1055.3$

 $a_{0y} = 1050.1 - (1.0110 \cdot 1055.3)$

 $a_{0y} = -16.8083$

(j) Standard estimate of error. Calculate a standard estimate of error, SEE, as follows:

$$SEE_{y} = \sqrt{\frac{\sum_{i=1}^{N} \left[y_{i} - a_{0y} - \left(a_{1y} \cdot y_{refi} \right) \right]^{2}}{N - 2}}$$
 Eq. 1065.602-11

Example:

N = 6000 $y_1 = 2045.8$ $a_{0y} = -16.8083$ $a_{1y} = 1.0110$ $y_{refi} = 2045.0$

$$\mathrm{SEE_y} = \sqrt{\frac{\left[2045.8 - \left(-16.8083\right) - \left(1.0110 \cdot 2045.0\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{\mathrm{ref}\,6000}\right)\right]^2 + \dots \left[y_{6000} - \left(-16.8083\right)$$

 $SEE_{v} = 5.348$

(k) Coefficient of determination. Calculate a coefficient of determination, r^2 , as follows:

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$$r_{y}^{2} = 1 - \frac{\sum_{i=1}^{N} \left[y_{i} - a_{0y} - \left(a_{1y} \cdot y_{refi} \right) \right]^{2}}{\sum_{i=1}^{N} \left[y_{i} - \overline{y} \right]^{2}}$$
 Eq. 1065.602-12

Example: N = 6000 $y_1 = 2045.8$

 $a_{0y} = -16.8083$ $a_{1y} = 1.0110$ $y_{refi} = 2045.0$ $\bar{y} = 1480.5$

$$r_{y}^{2} = 1 - \frac{\left[2045.8 - \left(-16.8083\right) - \left(1.0110 \times 2045.0\right)\right]^{2} + \dots \left[y_{6000} - \left(-16.8083\right) - \left(1.0110 \cdot y_{ref6000}\right)\right]^{2}}{\left[2045.8 - 1480.5\right]^{2} + \dots \left[y_{6000} - 1480.5\right]^{2}}$$

$$r_v^2 = 0.9859$$

(1) Flow-weighted mean concentration. In some sections of this part, you may need to calculate a flow-weighted mean concentration to determine the applicability of certain provisions. A flowweighted mean is the mean of a quantity after it is weighted proportional to a corresponding flow rate. For example, if a gas concentration is measured continuously from the raw exhaust of an engine, its flow-weighted mean concentration is the sum of the products of each recorded concentration times its respective exhaust molar flow rate, divided by the sum of the recorded flow rate values. As another example, the bag concentration from a CVS system is the same as the flow-weighted mean concentration because the CVS system itself flow-weights the bag concentration. You might already expect a certain flow-weighted mean concentration of an emission at its standard based on previous testing with similar engines or testing with similar equipment and instruments. If you need to estimate your expected flow-weighted mean concentration of an emission at its standard, we recommend using the following examples as a guide for how to estimate the flow-weighted mean concentration expected at the standard. Note that these examples are not exact and that they contain assumptions that are not always valid. Use good engineering judgment to determine if you can use similar assumptions.

- (1) To estimate the flow-weighted mean raw exhaust NO_X concentration from a turbocharged heavy-duty compression-ignition engine at a NO_X standard of 2.5 g/(kW·hr), you may do the following:
- (i) Based on your engine design, approximate a map of maximum torque versus speed and use it with the applicable normalized duty cycle in the standard-setting part to generate a reference duty cycle as described in §1065.610. Calculate the total reference work, $W_{\rm ref}$, as described in §1065.650. Divide the reference work by the duty cycle's time interval, $\Delta t_{\rm dutycycle}$, to determine mean reference power, $\bar{P}_{\rm ref}$.
- (ii) Based on your engine design, estimate maximum power, P_{\max} , the design speed at maximum power, f_{\max} , the design maximum intake manifold boost pressure, p_{inmax} , and temperature, T_{inmax} . Also, estimate a mean fraction of power that is lost due to friction and pumping, \bar{P}_{frict} . Use this information along with the engine displacement volume, V_{disp} , an approximate volumetric efficiency, η_{V} , and the number of engine strokes per power stroke (2-stroke or 4-stroke), N_{stroke} to estimate the maximum raw exhaust molar flow rate, \dot{n}_{exhmax} .
- (iii) Use your estimated values as described in the following example calculation:

$$\overline{x}_{\text{exp}} = \frac{e_{\text{std}} \cdot W_{\text{ref}}}{M \cdot \dot{n}_{\text{exhmax}} \cdot \Delta t_{\text{duty cycle}} \cdot \left(\frac{\overline{P}_{\text{ref}} + (\overline{P}_{\text{frict}} \cdot P_{\text{max}})}{P_{\text{max}}}\right)}$$
Eq. 1065.602-13
$$\dot{n}_{\text{exhmax}} = \frac{p_{\text{max}} \cdot V_{\text{disp}} \cdot f_{\text{nmax}} \cdot \frac{2}{N_{\text{stroke}}} \cdot \eta_{\text{V}}}{R \cdot T_{\text{max}}}$$
Eq. 1065.602-14

Example:

 $\begin{array}{l} e_{\rm NOX} = 2.5~{\rm g/(kW \cdot hr)} \\ W_{\rm ref} = 11.883~{\rm kW \cdot hr} \\ M_{\rm NOX} = 46.0055~{\rm g/mol} = 46.0055 \cdot 10^{-6}~{\rm g/\mu mol} \\ \Delta t_{\rm dutycycle} = 20~{\rm min} = 1200~{\rm s} \\ \bar{P}_{\rm ref} = 35.65~{\rm kW} \\ \bar{P}_{\rm frict} = 15\% \end{array}$

 $\begin{array}{l} P_{\rm max} = 125~{\rm kW} \\ p_{\rm max} = 300~{\rm kPa} = 300000~{\rm Pa} \\ V_{\rm disp} = 3.0~1 = 0.0030~{\rm m}^3 \\ f_{\rm nmax} = 2800~{\rm rev/min} = 46.67~{\rm rev/s} \\ N_{\rm stroke} = 4~1/{\rm rev} \\ \eta_{\rm V} = 0.9 \\ R = 8.314472~{\rm J/(mol\cdot K)} \\ T_{\rm max} = 348.15~{\rm K} \end{array}$

$$\dot{n}_{\text{exhmax}} = \frac{300000 \cdot 0.0030 \cdot 46.67 \cdot \frac{2}{4} \cdot 0.9}{8.314472 \cdot 348.15}$$

 $\dot{n}_{\rm exhmax}$ = 6.53 mol/s

$$\overline{x}_{\text{exp}} = \frac{2.5 \cdot 11.883}{46.0055 \cdot 10^{-6} \cdot 6.53 \cdot 1200 \cdot \left(\frac{35.65 + (0.15 \cdot 125)}{125}\right)}$$

 $\bar{x}_{\rm exp}$ = 189.4 µmol/mol

- (2) To estimate the flow-weighted mean NMHC concentration in a CVS from a naturally aspirated nonroad spark-ignition engine at an NMHC standard of 0.5 g/(kW·hr), you may do the following:
- (i) Based on your engine design, approximate a map of maximum torque versus speed and use it with the applicable normalized duty cycle in the

standard-setting part to generate a reference duty cycle as described in \$1065.610. Calculate the total reference work, $W_{\rm ref}$, as described in \$1065.650.

- (ii) Multiply your CVS total molar flow rate by the time interval of the duty cycle, $\Delta t_{\rm dutycycle}$. The result is the total diluted exhaust flow of the $n_{\rm dexh}$.
- (iii) Use your estimated values as described in the following example calculation:

$$\overline{x}_{\text{NMHC}} = \frac{e_{\text{std}} \cdot W_{\text{ref}}}{M \cdot \dot{n}_{\text{dexh}} \cdot \Delta t_{\text{duty cycle}}}$$
 Eq. 1065.602-15

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Example:

 $\begin{array}{l} e_{\rm NMHC} = 1.5~{\rm g/(kW\cdot hr)} \\ W_{\rm ref} = 5.389~{\rm kW\cdot hr} \\ M_{\rm NMHC} = 13.875389~{\rm g/mol} = 13.875389~\cdot~10^{-6}~{\rm g/} \\ \mu {\rm mol} \\ \dot{n}_{\rm dexh} = 6.021~{\rm mol/s} \\ \Delta t_{\rm dutycycle} = 30~{\rm min} = 1800~{\rm s} \end{array}$

$$\overline{x}_{NMHC} = \frac{1.5 \cdot 5.389}{13.875389 \cdot 10^{-6} \cdot 6.021 \cdot 1800}$$

 \bar{x}_{NMHC} = 53.8 µmol/mol

[70 FR 40516, July 13, 2005, as amended at 73 FR 37324, June 30, 2008; 75 FR 23044, Apr. 30, 2010]

§ 1065.610 Duty cycle generation.

This section describes how to generate duty cycles that are specific to your engine, based on the normalized duty cycles in the standard-setting part. During an emission test, use a duty cycle that is specific to your engine to command engine speed, torque, and power, as applicable, using an engine dynamometer and an engine operator demand. Paragraph (a) of this section describes how to "normalize" your engine's map to determine the maximum test speed and torque for your engine. The rest of this section describes how to use these values to "denormalize" the duty cycles in the standard-setting parts, which are all published on a normalized basis. Thus,

the term "normalized" in paragraph (a) of this section refers to different values than it does in the rest of the section.

(a) Maximum test speed, $f_{\rm ntest}$. This section generally applies to duty cycles for variable-speed engines. For constant-speed engines subject to duty cycles that specify normalized speed commands, use the no-load governed speed as the measured $f_{\rm ntest}$. This is the highest engine speed where an engine outputs zero torque. For variable-speed engines, determine the measured $f_{\rm ntest}$ from the power-versus-speed map, generated according to §1065.510, as follows:

(1) Based on the map, determine maximum power, P_{max} , and the speed at which maximum power occurred, f_{nPmax} . If maximum power occurs at multiple speeds, take f_{nPmax} as the lowest of these speeds. Divide every recorded power by P_{max} and divide every recorded speed by f_{nPmax} . The result is a normalized power-versus-speed map. Your measured f_{ntest} is the speed at which the sum of the squares of normalized speed and power is maximum. Note that if multiple maximum values are found, $f_{\rm ntest}$ should be taken as the lowest speed of all points with the same maximum sum of squares.

Determine f_{ntest} as follows:

$$f_{\text{ntest}} = f_{\text{ni}}$$
 at the maximum of $\left(f_{\text{nnormi}}^2 + P_{\text{normi}}^2\right)$ Eq. 1065.610-1

Where

 $f_{\text{ntest}} = \text{maximum test speed.}$

i = an indexing variable that represents one recorded value of an engine map.

 f_{nnormi} = an engine speed normalized by dividing it by f_{nPmax} .

 P_{normi} = an engine power normalized by dividing it by P_{max} .

Example:

 $\begin{array}{l} (f_{nnorm1}=1.002,\,P_{norm1}=0.978,\,f_{n1}=2359.71) \\ (f_{nnorm2}=1.004,\,P_{norm2}=0.977,\,f_{n2}=2364.42) \\ (f_{nnorm3}=1.006,\,P_{norm3}=0.974,\,f_{n3}=2369.13) \\ (f_{nnorm12}+P_{norm1}^2)=(1.002^2+0.978^2)=1.960 \\ (f_{nnorm2}^2+P_{norm2}^2)=(1.004^2+0.977^2)=1.963 \\ (f_{nnorm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3}^2+P_{norm3}^2)=(1.006^2+0.974^2)=1.961 \\ (f_{nnorm3}^2+P_{norm3$

(2) For variable-speed engines, transform normalized speeds to reference

speeds according to paragraph (c) of this section by using the measured maximum test speed determined according to paragraph (a)(1) of this section—or use your declared maximum test speed, as allowed in §1065.510.

(3) For constant-speed engines, transform normalized speeds to reference speeds according to paragraph (c) of this section by using the measured noload governed speed—or use your declared maximum test speed, as allowed in § 1065.510.

(b) Maximum test torque, $T_{\rm test}$. For constant-speed engines, determine the measured $T_{\rm test}$ from the power-versus-speed map, generated according to § 1065.510, as follows: